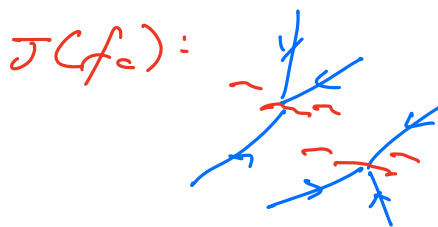
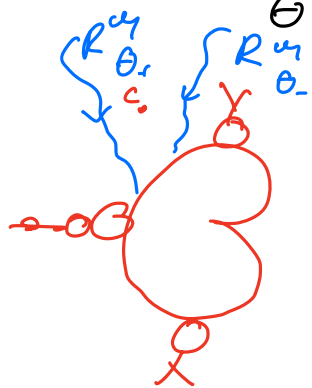
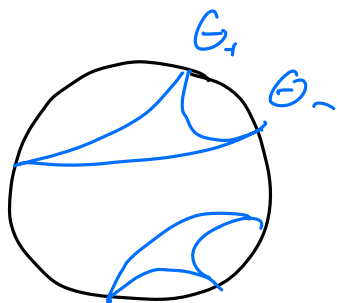


Rays, Roots, and Wakes

Recall 1) $c \notin M$, f_c has a postcritical A iff $\theta_c \in (\theta_-, \theta_+)$

2) $E^-(\theta) = \theta \Rightarrow R_\theta^u$ land on a periodic point $\leftarrow$ land at periodic



Thm. A postcritical is the interval (θ_-, θ_+)

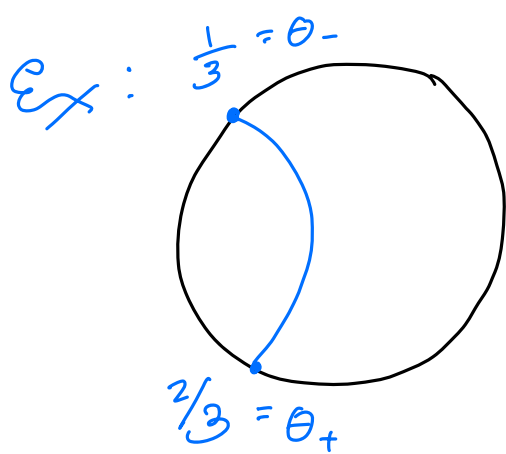
1) $R_{\theta_-}^u : R_{\theta_+}^u$ land at the same periodic point called the root r_A associated to A

$$\leadsto \mathbb{C} = \underbrace{W_A}_{\text{wake}} \cup (R_{\theta_-}^u \cup R_{\theta_+}^u \cup \{r_A\}) \cup (\mathbb{C} \setminus \overline{W_A})$$

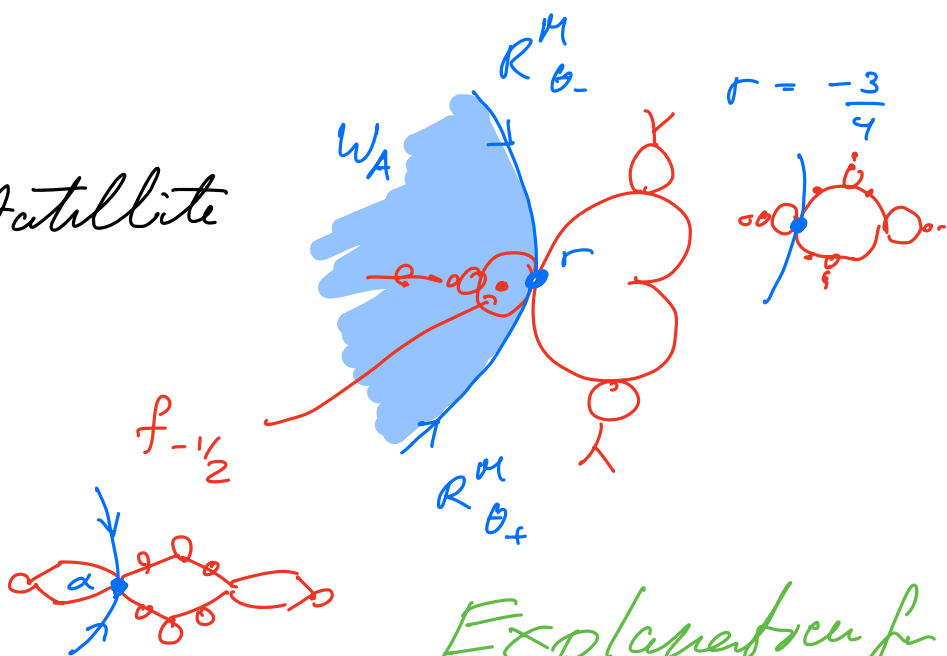
where W_A is s.t. $\mathbb{C} \setminus \overline{W_A} \neq \emptyset$.

2) f_c has a repelling cycle in postcritical A iff $c \in W_A$

Geography of the Mandelbrot

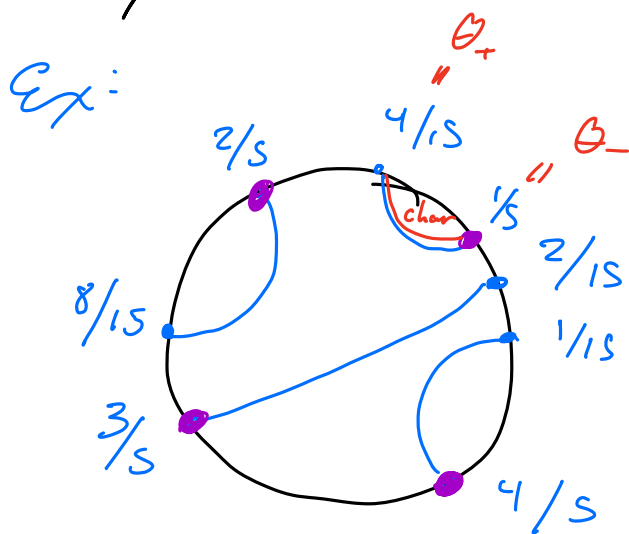


Satellite

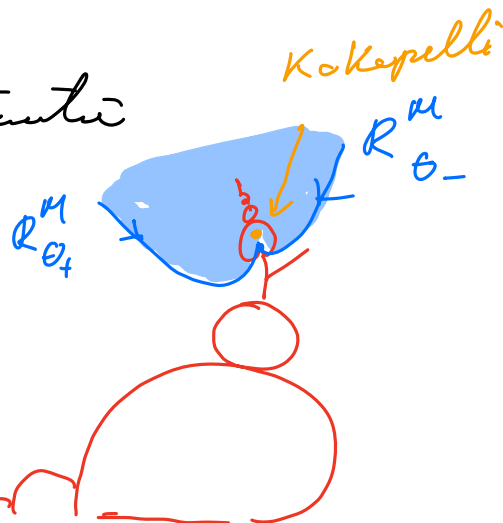


For anythg in the
make, and on the
2-figl pt.

Explanation for
slender geometry

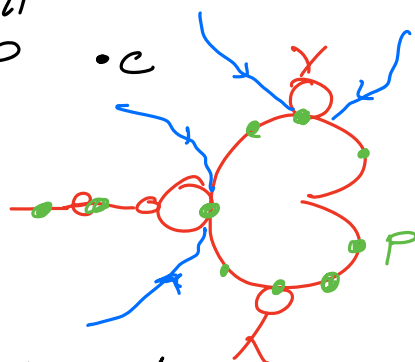
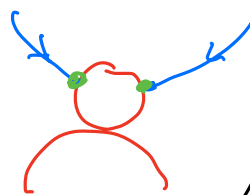


Perturbative



Proof. 1) $X = \bigcup_{\theta \in UA_i} R^{\theta}_\theta \cup \{ \text{pericelli u-green} \}$

Goal: $R^{\theta}_\theta, R^{\theta}_\theta$ lead at the



Eqns, want the map to descend $\mathbb{C}$.
if $c \notin X$.

$\perp c \in \mathcal{M}$: all maps $R_{c,\theta}$ if $\theta \in UA_i$

lead at repelling pts and none too nearby

— $c \notin \mathcal{M}$: all rays $R_{c,\theta}$ of $\theta \in UA$; lead set

repelling pts and none too ^{nearly} $V_{\text{sin}} E^u(\theta) \neq \theta_c$

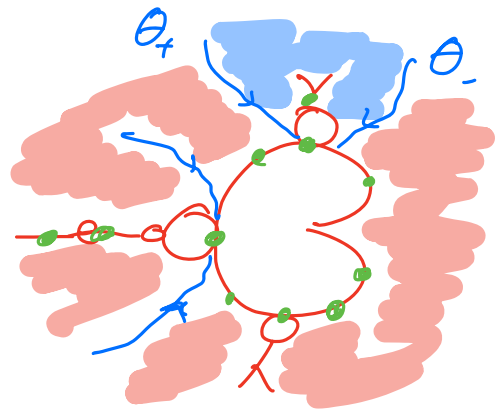
$\Rightarrow \mathcal{O}_c$ is every component of $\mathbb{C} \setminus X$, leading pattern of the rays is the same.

We have that for $c \notin \mathcal{M}$, f_c has a rep. cycle iff

$\theta_c \in (\theta_-, \theta_+)$. The only

ray to disconnect $\{c \notin \mathcal{M} \mid \theta_c \in (\theta_-, \theta_+)\}$

from $\{c \notin \mathcal{M} \mid \theta_c \notin [\theta_-, \theta_+]\}$ is for the rays to lead at the same pt.



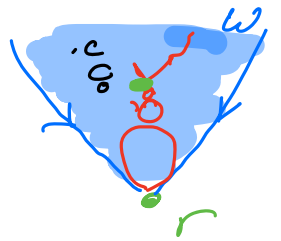
2) f_c has a repelling orbit by point A

if $c \in W \setminus P$

f_c does not have a rep. orbit by pt A if

$c \in \mathbb{C} \setminus (\bar{W} \cup P)$

What about P ?

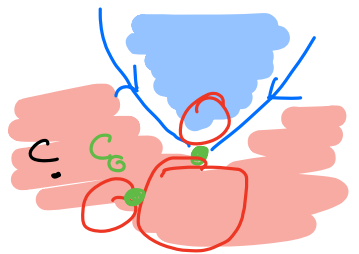


$c_0 \in W \cap P$: $c \in W \setminus P$, f_c

has a sup. cycle, $R_{c, \theta_{\pm}}$ land at the same pt w mult $\lambda(c)$. Take $c \rightarrow c_0$, $R_{c_0, \theta_{\pm}}$ will land at the same pt.

of $J(f_{c_0})$. WTS it is repelling:

$\lambda: W \rightarrow \mathbb{C}$ is hol, $|\lambda(c)| \geq 1$ for $c \in W \cap P$.
 $|\lambda|$ can't have a local min at $c_0 \in W$,
 so f_{c_0} has a repelling cycle.

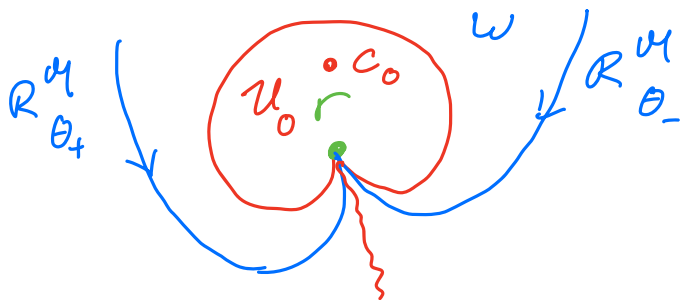


• If $c_0 \in P \setminus W$ has a repelling orbit, then by perturbing it $\exists c \in \mathbb{C} \setminus (W \cup P)$ we would get a sup. cycle as part of A' in $J(f_c)$, but can't.

Thm. Pertinent $A = \{A_1, \dots, A_k\}$, $\cup A_i$
 for orbit of period n

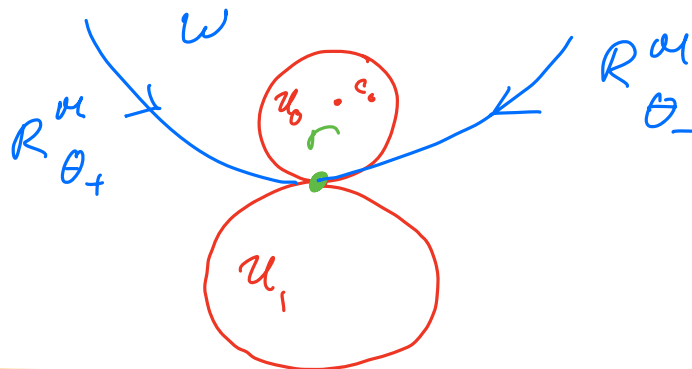
Primitive

$|A_i| = 2$, 2 cycles, $n = K$



Satellite

1 cycle, $n = K \cdot |A_i|$



#2 $R^u_{\theta_{\pm}}$ land on r ,
potential of the
possible cycles of f_r
in A

#1 $\exists!$ hyp. comp U_0 s.t.
 $r \in \partial U_0$, $U_0 \subset W$

#4 ∂U_0 is a connected arc • ∂U is smooth
comp $d r$

#1 • If $c \in U_0$, f_c has an
attr. cycle of period $n = K$

#5 $\exists! c_0 \in U_0$, center, s.t. f_{c_0}
is superattracting

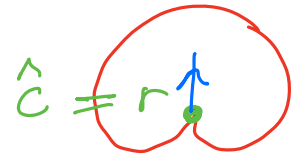
#3 $\exists!$ hyp. comp U_1 s.t.
 $U_1 \subset \mathbb{C} \setminus W$, $r \in \partial U_1$

#4 • ∂U_1 is smooth

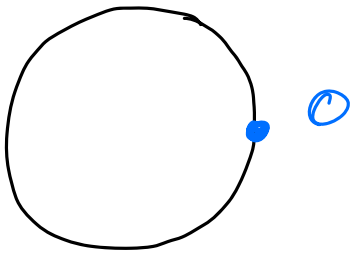
#3

• if $c \in U_1$, f_c has an attr. cycle of period K

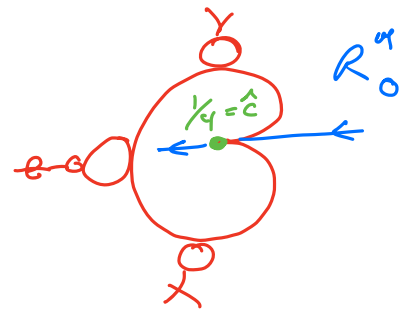
Lemma 1. Let $\hat{c}$ be periodic in mult $\hat{\lambda}$,
perturb A . Then $\exists c(t)$, $t \in [0, \varepsilon]$
s.t. $c(0) = \hat{c}$, $f_{c(t)}$ has a rep
cycle of period K in perturb
 A , and an attracting cycle of per. K . As
 $c(t) \rightarrow \hat{c}$, the two cycles approach the rep
cycle of $f_{\hat{c}}$.



Ex: Parameter: $A = \{ \{0\} \}$, $K = 1$ (Except)

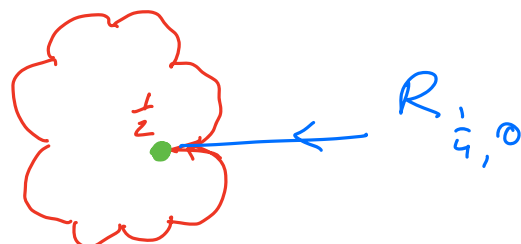
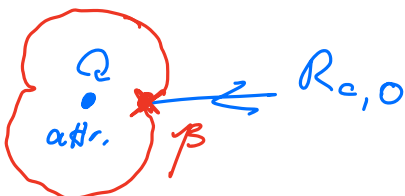


For $c < \frac{1}{4}$, f_c has
an attr. fixed pt and
a rep. fixed pt.

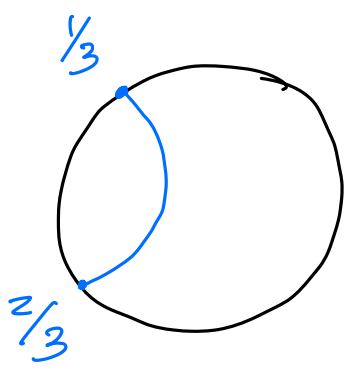


$$c < \frac{1}{4}$$

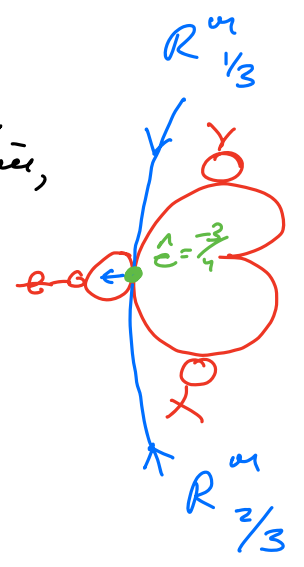
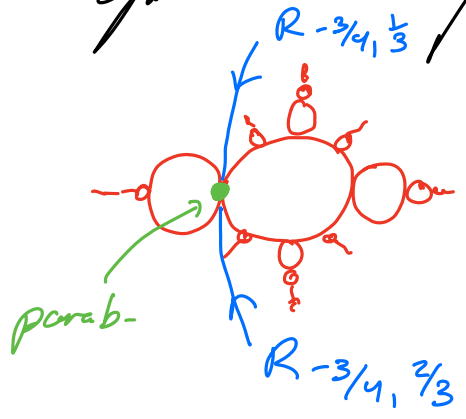
$$c \rightarrow \hat{c} = \frac{1}{4}$$



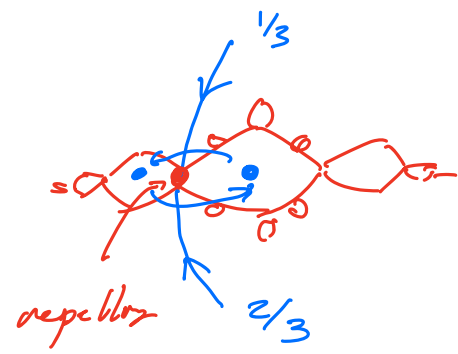
Ex: Satellite: $n=2, k=1$



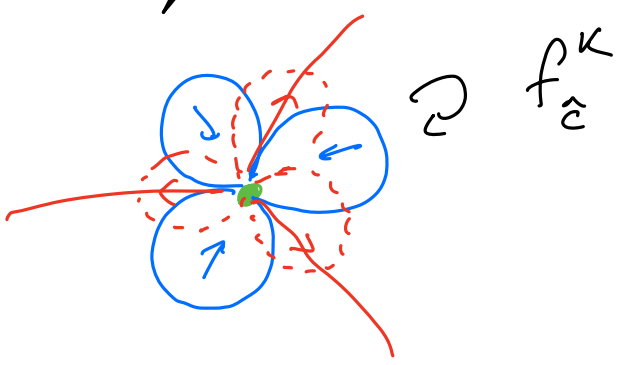
For $c = -\frac{3}{4}$, jet direction,
parabolic fixed pt.



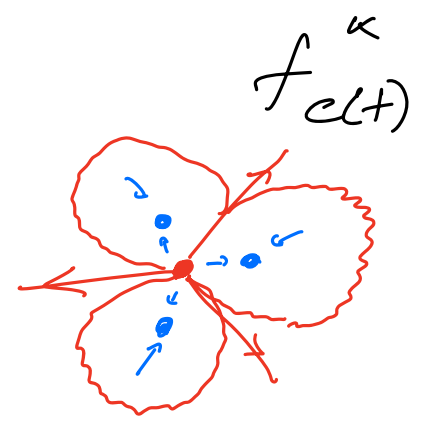
For $c < -\frac{3}{4}$, f_c has an elliptic cycle of per $u=2$,
rep fixed pt. $c = -1$: Braster



Sketch. u rays lead at the period of f_c^u ,
have period r Prim: $r = k$ Set: $r \leq u = k \cdot |A|$



$\rightsquigarrow$



Under f_c^k , it takes r/k to rotate, so $(\sum^k)^{r/k} = 1$
 $\Rightarrow r$ th root of unity

Choose $c(t)$ so that $\lambda_{c(t)}^r = 1+t$, repelling
The repulsion/attraction in the patch
will persist.

↳ Schwere $\Rightarrow$ Atte. geht besser aus.
Lern.

↳ External rays continue to land

Cor. #2 $\mathcal{C}$ primal $\Rightarrow c$ is the cost when $R_{\Theta_{\pm}}^{01}$
 dual for the primal Δ for $f_{\mathcal{C}}^1$

Proof. $\hat{C} \in \mathcal{I} \mathcal{U}_0$, $\mathcal{U}_0 \subset \mathcal{W}_A$. But

$\hat{c} \notin W_A$ since it is proved. by postulated A, not
exp. by postulated A. $\Rightarrow \hat{c} \in 2W_A \cap M$
" $\{v\}$.

Lemma. #3 Let $\hat{c}$ be possible;

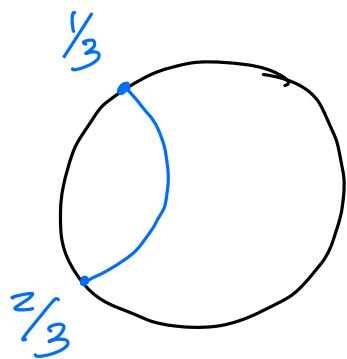
let A be the postulant of the prob. exch.

Suppose A is satellite. Then $\exists$ path.

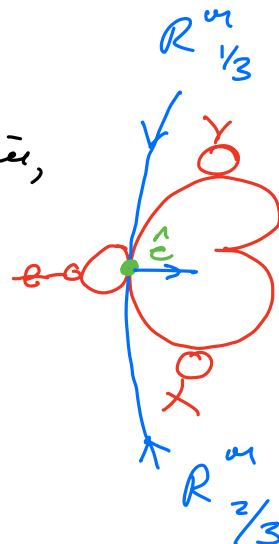
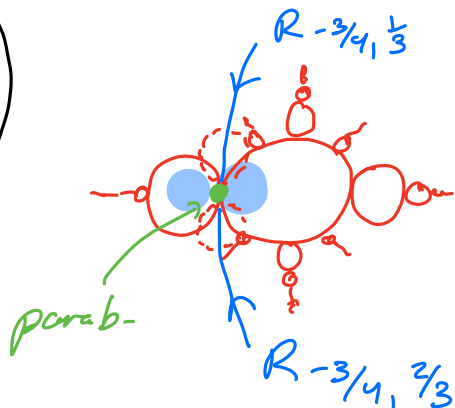
$$c(t), t \in [0, \varepsilon], \text{ st. } c(0) = \hat{c} \text{ and } f_c(t)$$

has an attr. orbit of period K and a
rep. orbit of period n as partent $\{\bigcup_{i=1}^n A_i\}$.

Ex: Satellite: $n=2, k=1$

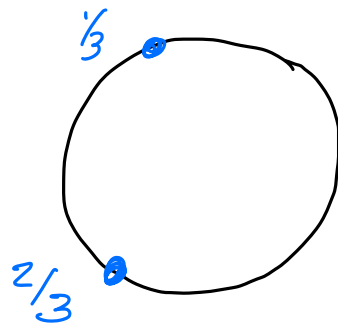
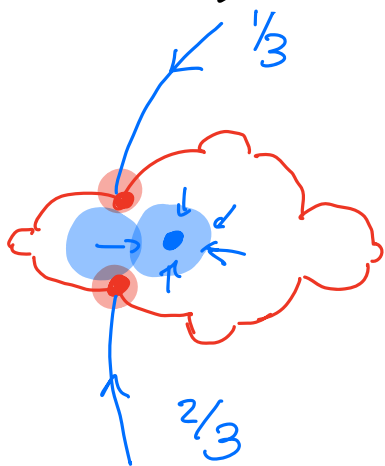


For $c = -\frac{3}{4}$, first iteration,



For $c > -\frac{3}{4}$, f_c has an attr. fixed pt and a rep 2-cycle as potential $\{\frac{1}{3}\}, \{\frac{2}{3}\}$.

$J(f_c)$:



Sketch. Pick $c(t)$ s.t. $\lambda_{c(t)}^r = 1 - t$:

repulsive $\leadsto$ attract, attracty pt $\leadsto$ attr. basin
repelly pt w/ persist $\leadsto$ repelly cycle.